LA-UR-09-05016

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Intended for: LANL Report

LANL* V2.0: A RADIATION BELT DRIFT SHELL MODEL

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3 August 2009

Abstract

We present here a new method for calculating L* (L-star), the magnetic drift invariant. L* is used extensively in modeling radiation belt dynamics and many other space weather applications. L* is proportional to the integral of the magnetic flux through a surface whose boundary is the path of a charged particle moving through the Earth's geomagnetic field. The equations of motion that describe very energetic particles in Earth's radiation belts are most commonly expressed in terms of L*, energy (momentum), and pitch angle (the angle between the particles motion and the magnetic field lines). The use of L* helps simplify the equations of motion. However, a typical L* calculation can require on the order of 10^5 calls to a magnetic field model. To simplify these calculations, we have developed and tested a neural network surrogate model to calculate L* values with low induced error and computational speeds millions of times faster than direct numerical integration. We used the Tsyganenko 2004 magnetic field model for our calculations. This is the most recent of a series of models published by Tsyganenko and colleagues. To train and validate our model, we used a collection of solar wind data amassed over the past four and a half decades. We refer to this numerical application for calculating L* as the Los Alamos National Laboratory L* model (LANL*).

Introduction

The motion of charged particles in a realistic, complicated geomagnetic field can be closely modeled using “guiding center” theory. This theory represents the motion of the particle as a function of three adiabatic invariants - μ, K, and L^*. The first two invariants are relatively simple to compute because they involve single integrals along a single field line. The invariants μ and K are measures of gyro and bounce motion, respectively. The gyro invariant, μ, measures the particle's motion around a magnetic field line and is on the order of milliseconds. The bounce invariant, K, is a measure of the particle's motion parallel to the magnetic field lines and has a time scale on the order of seconds. The latter, L^*, is much more complicated to compute because its integral is multidimensional and it spans the entire 3D space. A typical L^* computation involves on the order of 10^5 calls to the magnetic field model. Being computationally intensive, researchers often resort to using simpler magnetic field models to speed up computations at the expense of reduced accuracy. In this project, we implemented a neural network based surrogate model that accurately reproduces the same results as a direct numerical integration for calculating L^*. We call this application of surrogate models the Los Alamos National Laboratory L^* model, or LANL* for short.

Background

The equations of motion that describe highly energetic particles in the Earth’s radiation belts are typically expressed by diffusion in three dimensions – L^*, energy, and pitch angle (the angle between the velocity vector and the magnetic field vector). Expressing motion in these coordinates greatly simplifies the problem by referencing particle distribution functions to values in the magnetic “drift shell” disregarding time or longitude. While utilizing L^* greatly simplifies the equations of motion, it is also useful in other practical applications such as space weather forecasting. The problem with using L^*, as we have already described, is that it is computationally expensive. The amount of time used in directly computing L^* values can be more of a drawback to researchers than the inaccuracies of simpler magnetic field models.

L^* is defined as:

$$L^* = -\frac{2\pi k_0}{\Phi R_E}$$

Here, k_0 is the Earth’s dipole moment, R_E is the radius of the Earth, and Φ is defined as:

$$\Phi = \int \boldsymbol{B} \cdot d\,\boldsymbol{S}$$

In a dipole magnetic field, L^* is the distance from the center of the Earth to the equatorial point of the magnetic field line in question, in units of Earth radii. Computing L^* in a dipole field would be relatively simple, but wouldn’t give an accurate depiction of the Earth’s magnetic field. The Earth’s magnetic field is constantly being reshaped by the solar wind and magnetospheric and ionospheric current systems. These factors create such significant distortions in the Earth’s magnetic field that it can no longer be approximated by a magnetic dipole, and therefore require much more complex models that require more time to analyze and perform calculations. It is for this reason that researchers are seeking alternatives to calculating $L*$. In our project, we develop one such model utilizing a neural network based surrogate model.

Method

Surrogate models can replace complicated, non-linear input-output relationships while introducing virtually insignificant error. They consist of multiple input nodes (one for each parameter), hidden nodes, and an output node (Figure 1). These nodes are similar to neurons in our bodies. Our surrogate model was trained using input-output data from a training set computed by the standard numerical integration technique. Surrogate models do not consider the details of the calculation. Instead, they focus on the input-output relationship. Results from surrogate models are not exact, but can be made to have an arbitrarily small inaccuracy depending on the number of training samples and hidden nodes. Barron's (1991, 1993, 1994) study showed that error decreases proportional to $1/\sqrt{N}$ and $1/M$, where N is the number of training samples and M is the number of hidden nodes. However, with too many hidden nodes, the network may simply memorize patterns, which can lead to inaccurate results.

For our model, we used a feedforward neural network. Artificial neural networks closely approximate the method of our nervous systems by representing a non-linear mapping of input-output signals (Bishop, 1995; Reed and Marks, 1999). The hidden nodes are interconnected with the input nodes and the output node. Feedforward neural networks do not allow signals from output to cycle back to input. This results in faster processing time. Each node has weighted connections. During training, the weights are adjusted in order to produce the desired results. Using this method, we have developed a surrogate model for computing $L*$ values with a typical error of less than 1%. Our neural network can calculate $L*$ in a fraction of the time it takes for direct drift shell integration. Half a million calculations can be performed in seconds – a great improvement over the 1700+ hours it would take with direct methods.

We validated our network by comparing its results to those obtained by the standard numerical integration method. Figure 2 shows the $L*$ values computed with the standard method of integration compared to those obtained with the neural network. Notice the relatively small error as compared to the $L*$ values. Figure 3 presents the size of the error in histogram form to give a better visual picture of the small error induced when using the neural network method. For the training data, we used data obtained from various LANL geosynchronous satellites. We calculated their $L*$ values in hourly increments and compared to our network's values.

Conclusion and Summary

$L*$ is a valuable variable in that it helps to simplify the equations of motion for particles in geomagnetic fields. The drawback to using $L*$ is that is time consuming to compute. Using a neural network based surrogate model, we have developed a method for computing $L*$ values orders of magnitude faster than the standard integration methods. This method will be very valuable to researchers working in space physics as $L*$ useful in forecasting space weather. By using a feedforward network, it is possible to greatly decrease computing time without introducing more than 1% error.

Figures

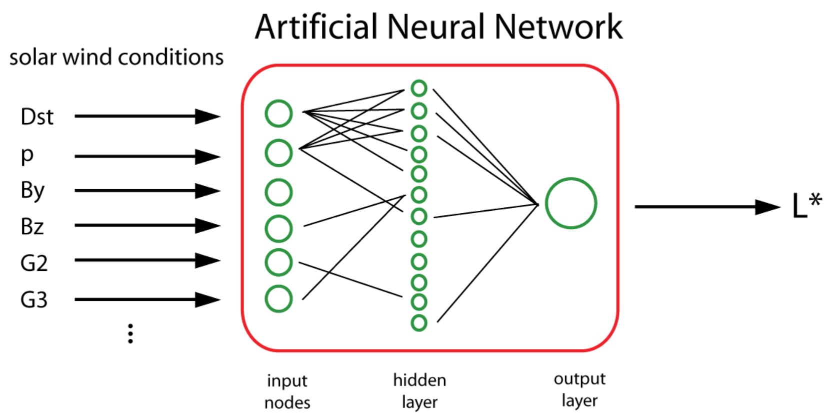

Figure 1. Schematic diagram of a neural network. The input values enter through the input nodes on the left, pass through the hidden layer, and finally emerge from the output layer giving the value of L^*. The connectors (represented by lines) are weighted during training as to produce the desired results with minimal error.

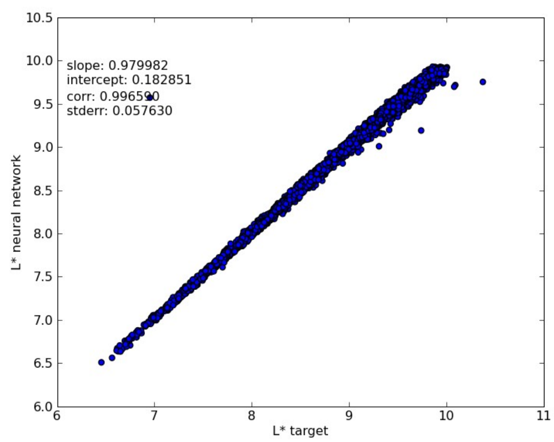

Figure 2. Neural network L^* values compared to standard integration method. The x-axis holds the values obtained through direct integration and the y-axis holds the values obtained with the neural network. Note the relatively small error of $\Delta L^* \sim 0.06$ compared to the L^* values ranging between 6 and 11. This give an error of less than one percent.

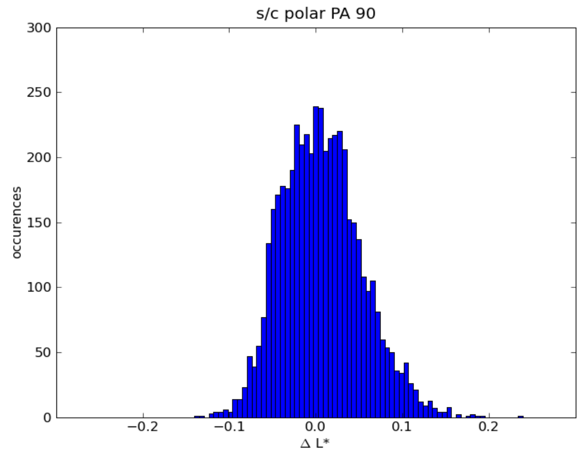

Figure 3. Error in *L**. The x-axis holds the error in *L** and the y-axis holds the number of occurrences for each magnitude of error. Again, note the relatively small error compared to the *L** values.

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